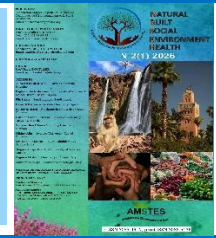




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Rare Events: Are Outliers Negligible?

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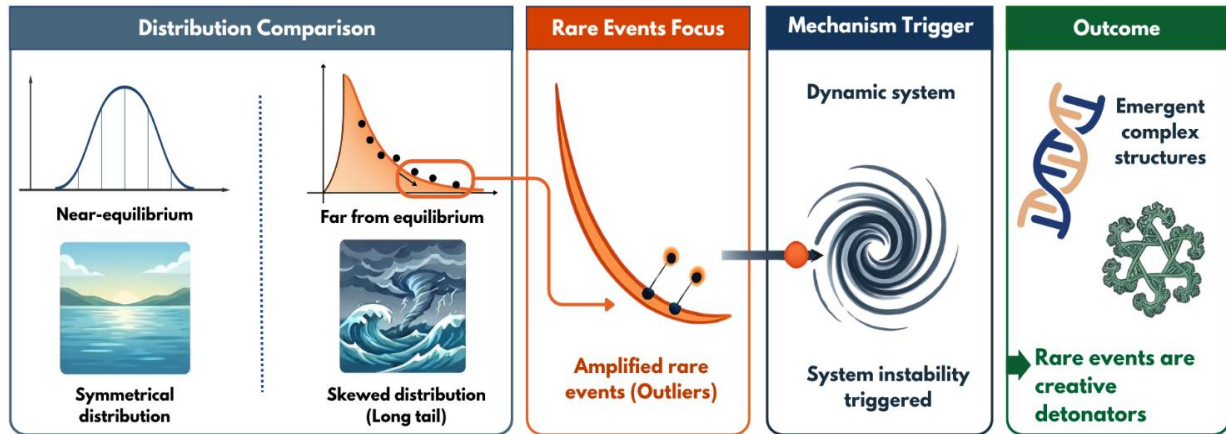
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Graphical Abstract

Do rare events sit at the margins, or do they shape the course of complex systems? As density falls and symmetry breaks, these overlooked “outsiders” emerge, revealing how fluctuations and asymmetry reorganize behaviour beyond the average. Far from equilibrium, such rare trajectories are amplified, triggering instabilities that quietly steer systems toward new, often unpredictable paths.



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Abstract: Do rare events merely sit at the margins of statistical distributions, or do they shape the evolution of complex systems? This work explores how asymmetry and fluctuations, especially in non-equilibrium conditions, redefine system behaviour. Numerical simulations show that as particle density decreases, distributions become increasingly skewed, revealing the growing role of extreme, low-probability configurations. These “outliers”, often overlooked in favour of mean behaviour, emerge as dynamically distinct components. Through entropy analysis and simple interaction models, the study demonstrates how local fluctuations and initial heterogeneities can lead to structured, asymmetric organizations. These effects extend beyond physics, offering insight into biological and systemic transformations driven by rare configurations. A simplified electro-driven molecular model further illustrates that rare trajectories can trigger nonlinear instabilities, leading to trapping, convection, and enhanced transport. In such regimes, fluctuations are amplified rather than damped, reshaping global dynamics. Overall, rare events are not negligible anomalies but active drivers of system evolution, steering complex systems beyond predictable averages.

Keywords: *Rare events, Asymmetry, Dynamics of out-of-equilibrium systems.*

Time's Arrow and Thermodynamics: A Taboo?

The question of time's direction has sparked endless debates and will likely continue to do so. More precisely, its link to thermodynamic irreversibility deserves close scrutiny. Although time is not explicitly part of thermodynamics' laws, the second law states that every process must increase global entropy. Jill North [1] explains that in a system like a gas of randomly distributed particles—macroscopically described via Boltzmann's statistical framework—extreme fluctuation domains can show localized entropy decreases, even as the system's overall entropy rises. Thus, time's evolution towards higher entropy does not rule out regions of major deviations¹ where disorder (entropy) is greatly reduced. According to Harold Blum in *Time's Arrow and Evolution* [2], the inexorable thrust of time—embodied in the second law of thermodynamics and its decree of rising entropy—serves as the cosmic compass guiding life's unfolding drama. From the universe's primal forge, physical and chemical edicts have carved narrow channels for evolution, curtailing possibilities long before life's tentative spark ignited.

This shifts us from time's arrow to what rarity, grasped in time, truly adds—or subtracts—from the extraordinary. First, what do we call a “rare event”? Common sense says it is any event with a near-zero probability. Yes—but who is asking?

Imagine a fund manager handling Euromillions: twice weekly, 104 times a year, he transfers 20–200 million euros (or the equivalent) to some winner. This has been routine since February 2004, when the Lotto game was launched.

Now picture a retired winemaker from Lézignan-Corbières winning 70 million. Life and family are upended—how could they foresee being the first winner in their department after 22 years? Odds: 1 in 113,983,816!

Does this not prompt reflection on what one sees as rare or improbable? Perspective matters.

In the Big Bang model, as the early universe evolved, tiny initial variations in particle densities were amplified by gravitational collapse—a self-reinforcing process that pulls in more and more particles, forming increasingly dense clumps. Similarly, the Janus model [3] suggests that “negative” and “positive” particles attract those of the same type and repel opposites, sparking the creation of both densely packed regions and vast empty voids (such as the Great Repeller). These mechanisms show how systems can develop areas with extreme deviations, whose traits starkly

¹ supporting therefore very unlikely trajectories.

contrast with the overall probabilistic system, where fluctuations remain small and near the average. Some even propose that Earth—or any local structure, like a tree leaf or a human body—could be such a region, evolving towards lower entropy relative to the whole universe. This essay explores to what extent systems, like cooling gases, can produce these domains, and whether such rare small-scale setups might give rise to highly improbable complex structures. Anisotropy, fluctuations, and emergence arise together as seeds of the dynamics we observe in the universe. Physics teaches us that near a critical point—like the phase transition from liquid to gas—fluctuations become macroscopic and pivotal: a small one can take over the entire system, reshaping its global properties. It often seems the same holds true (at times? often?) for matter and human affairs

Tails of Fate: How Pareto Forges Asymmetry

To simulate a cooling gas, we use a simplified numerical model. This model features an under-sampled universe of constant volume (represented as a 2D surface) where particle density decreases as the universe expands and cools. The simulation has two parts:

- Non-interacting particles: no interactions (e.g., gravitational forces) are considered.
- Mass–mass interactions: a basic approximation of particle–particle attraction is introduced.

In the first part, the simulation iteratively generates random particle positions using a pseudo-random number generator. The number of particles N varies logarithmically from 30 to 10,000 (10^4), with 100 discrete steps. For each N -particle setup, 30 random iterations calculate nearest-neighbour distances, with the random seed changed each time. The skewness² of the resulting spatial distribution is averaged over iterations, and the standard deviation is computed. We then analyse the evolution of skewness versus particle density³.

In the second part (see Appendix), the system is modelled on a 32×32 grid. Temporal entropy evolution is quantified using a simplified mass–mass attraction scheme (details below).

The first simulation's result appears in Figure 1, showing skewness evolution as particle density drops—as in universe expansion and cooling. The arrow of time points rightwards on the graph. Red dots show averages ($n = 30$) $\pm$ standard deviation in blue. We first note that average skewness

² *Skewness* indicates whether the data are spread out more on the left or on the right side of the average, instead of being evenly distributed.

³ The simulations are run in the R statistical environment.

stays above 0.5 in all cases, even though a large population ($N \gg 0$) should trend towards a normal, purely Gaussian distribution (the perfectly symmetric bell curve centred on its mean), with zero skewness. This stems from the lower bound constraint on particle encounters: distance can be zero but not negative.

As particle density falls from 10^4 to 10^3 , skewness rises, along with the spread of values, as shown by the growing standard deviation (blue bars in Fig. 1). Overall, this means that “outliers”—particles in sparse space—become proportionally more common relative to the total number of particles at low density, with skewness rising exponentially at the lowest densities, especially for $N = 30$ in this case: at very low density, skewness exceeds 1, a value typically signalling a highly skewed distribution

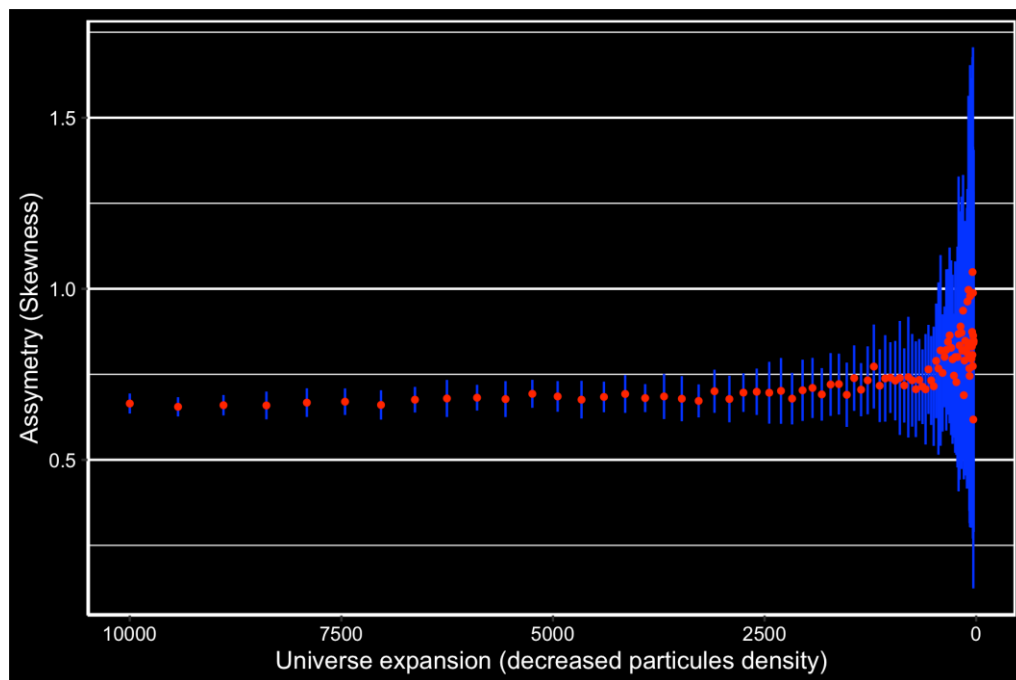
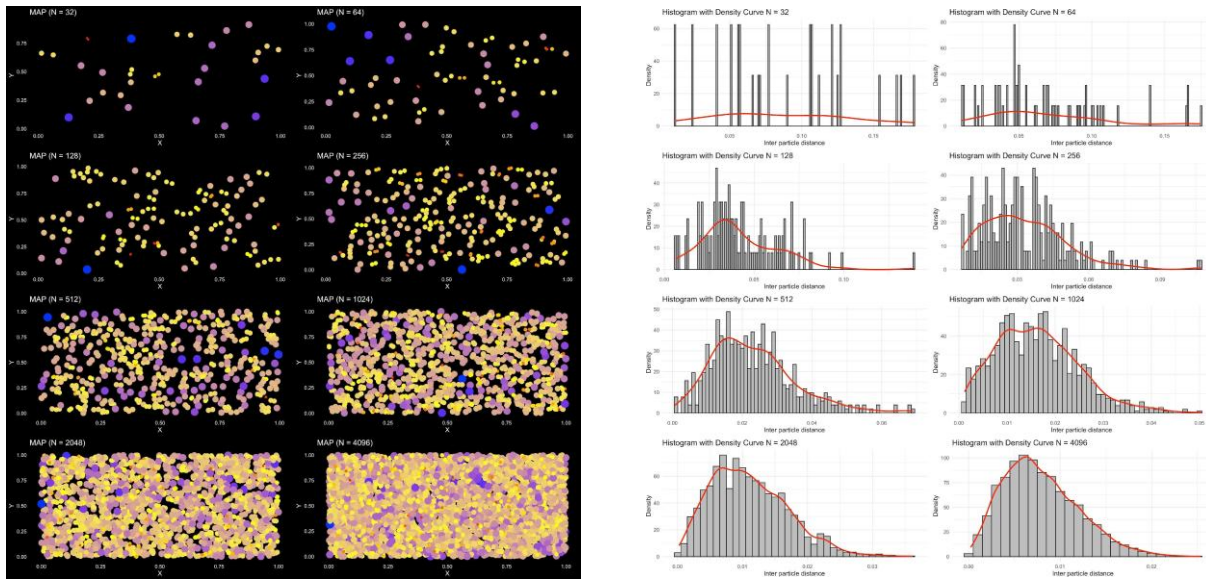


Figure 1. Evolution of skewness as particle density decreases

To illustrate better skewness evolution as particle density changes, Figure 2 shows spatial distributions and histograms of nearest-neighbour distances for random simulations with $32 \leq N \leq 4096$. Density curves (in red in Fig. 2b) help to visualize the asymmetry of the corresponding distributions. For example, skewness rises from 0.67 to 1.11 as the number of particles N drops from 1,000 to 100. Note that the parameter here is the minimum distance between two particles. In Figure 2a, particle colour and size encode their nearest-neighbour

distance: smaller size and redder colour mean that particles are closer to another particle; larger size and bluer colour mean that particles are farther from all others.



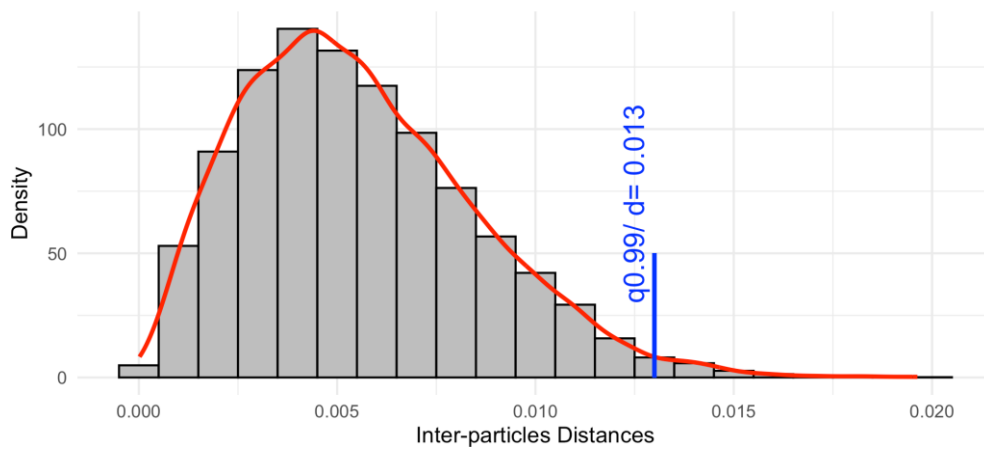
(a) Random spatial distributions

(b) Nearest-neighbor distributions

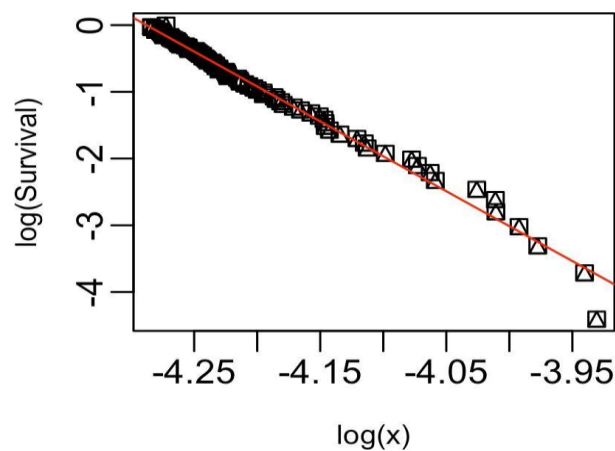
Figure 2. Initial distributions ($32 \leq N \leq 4096$)

To characterize better the long tail of the nearest-neighbour distance distribution, we analysed the case $N = 4096$, $n = 8192$, which yielded a skewness of 0.686. We tested whether this distribution exhibits a heavy tail that can be fitted by a Pareto law, which would more precisely describe its asymmetry. The data were analysed using the R package ExtremeFit [5]. This package first identifies an adaptive threshold τ beyond which the data appear to follow a Pareto distribution, then tests the goodness of fit for the data points beyond this extreme quantile ⁴.

⁴ The hill.adapt function returns $\tau = 0.0139$ (HASXadapt), the literature's adaptive threshold from which the tail can reasonably be modeled as Pareto-distributed



(a) Minimal distances distribution



(b) log–log plot for the 0.99 extreme quantile

Figure 3. Pareto evaluation for $N = 4096$ and $n = 8192$

Thus, Figure 3a shows the 0.99 quantile of this distribution, indicating that a second approach was developed to quantify entropy variations over time (Appendix A).

What Have We Learned So Far?

Do outliers matter? That is the question this essay aims to answer, with some thoughts on their possible consequences. In science, what catches the eye in an observed distribution is usually its first two moments: the mean and standard deviation (their position and uniformity). Tails—left or right—are often overlooked and attributed either to measurement errors or to individuals not worth a second glance. One exception: security services, which actively zoom in on rare, unexpected human behaviours spotted by surveillance cameras— “the exception” becomes the algorithm’s

target! The perspective shifts when we ask how these outliers evolve over time compared to those near the mean, for example within two standard deviations and thus representing 95% of the population ($\bar{x} \pm 2\sigma$). Do the “mainstream” truly rule the world? Should we pay attention to rare events?

In this simulation, a particle may represent a simple gas molecule or a small world, small compared to the size of the system, which is vastly larger than that of the “particle” itself, whatever its representation or kind⁵... At all time scales, gravitational forces or any other type of interaction are responsible for the spatial fluctuations of the particles, whether or not they collide. As mentioned above, any distribution observed at a given instant t serves as the starting point for the subsequent dynamics. The origin of the time arrow is itself dynamical! It is well known that very similar initial conditions can later give rise to drastic divergences, characterized by the Lyapunov exponent⁶. This is because the probability of a particular event, for example rolling a six on a die, does not depend on the previous roll⁷.

Since this approach stems from asymmetric distributions—even when $N \gg 0$, which is a priori unexpected—we must consider both the “left” and “right” tails of the density curves. Here, skewness is always positive and above 0.5; asymmetry is thus more pronounced on the “right” side than on the “left” side of the bell curve. This means that particles with large distances to their nearest neighbours outnumber those forming clumps. We shall call these the “right” and “left” cases. The “right” case—the famous heavy tail—groups particles (or tiny solitary worlds) that interact weakly with the system, since interactions between two entities classically decay as the inverse square of their separation distance. This means that they are barely exposed to external forces, contributions, or noise, evolving quasi-autonomously—based mainly on their internal forces⁸.

At the opposite end, the “left” case—less populated but real (see Fig. 2a), especially in a hot universe—exists. Unlike the previous case, these particles or small worlds face external forces from nearby neighbours: they act on their closest neighbours and at the same time are exposed to their

⁵ Here a “particle” is understood as an object without any particular spatial extent.

⁶ A Lyapunov exponent measures how quickly two trajectories of the same system, starting from nearly identical initial conditions, separate from (or approach) each other over time.

⁷ In mathematics this is the opposite of Markov chains observed in many different contexts ; one then speaks of independent events.

⁸ We consider here that a particle can represent a complete little world of its own, subject to its own laws regardless of its nature, a sort of scale-free entity.

effects. This status is the opposite of that of distant particles or worlds. Picture the galactic centre, where stellar density is huge: Proxima Centauri, the Sun's nearest star, lies 4 light-years away, whereas at M32's core—the “compact dwarf galaxy”—stars are just 3 light-days apart! We must admit that these two types of subsystem show very different dynamics

These distinct dynamics for both statuses can be illustrated, in my view, by what's called conformational drift in recombinant proteins [6]. These proteins are “artificially” synthesised by bacteria or yeast after targeted genome modification (GMOs ⁹). Specifically, these are dimeric proteins—made of two distinct polymeric chains that assemble post-synthesis to become biologically functional. If recombinant chains are synchronously synthesized, they assemble correctly. Conversely, if a time lag exists between the two chains' synthesis, the first begins relaxing: its conformation evolves through interactions with solvent molecules, its nearest neighbours. As a result, the second chain—synthesized later—no longer recognizes its partner, and the functional dimer never forms. This is called conformational drift. Such protein misfolding causes many genetic diseases, including cystic fibrosis.

Let us return to high-deviation regions. It is very likely that they exist and evolve differently depending on whether they form a sparse environment (“right” case) or a highly dense one (“left” case). One may wonder whether such distinct states have been observed in the universe. In fact, in the core of certain galaxies, the matter density is about $3 \times 10^{-14} \text{ kg/m}^3$, whereas in cosmic voids it drops to approximately 10^{-22} kg/m^3 . This eight-order-of-magnitude difference reflects, as mentioned above, the evolution of density fluctuations over time as the universe expands.

At this stage, one must admit that skewness arises spontaneously, and that large deviations can exist and evolve within the global system, without violating the probabilistic interpretation of the second law of thermodynamics¹⁰. Although they evolve towards opposite entropy values, their contributions to the global macrosystem remain, apparently, negligible given their rarity and their tiny spatial scale. We shall see that the time evolution of any system is governed by its network of interactions: even an infinitesimal initial difference can lead over time to systemic divergences that appear alien with respect to the dominant behaviours. Yet, these observed divergences ultimately stem from the intrinsic dynamics of out-of-equilibrium systems. If such microscopic worlds wished

⁹ Genetically Modified Organisms.

¹⁰ In fact, the second law of thermodynamics is not violated, since it applies only to macroscopic systems and not to micro sub-systems.

to communicate with similar entities, they would require infinite patience and the development of extraordinarily powerful, extremely long-range radio telescopes.

The Paradox of Infinite Improbability

Before discussing the role of these high-deviation regions in system and subsystem dynamics, let us quickly survey rare events that occur despite seeming nearly impossible. The “historical” field is full of such cases. According to Hannah Arendt [7], every human action appears as a miracle—that is to say, an infinite improbability that repeats. Beyond its own infinite improbability, the power of even the smallest event lies in its ability—even amid the most constrained circumstances—to change all combinations of circumstances. In modern history, or “modern science”, events arise unexpectedly, without roots in hidden underground currents. Historians strive to weave a “logical narrative” to explain them, yet no causal link fully accounts for their occurrence.

The most striking example is the origin of life. Hannah Arendt gets straight to the point: it is the infinite improbability of the event that breathes life into what we call the real. Following Arendt [7], the origin of life in matter may be understood as an expression of infinite improbability arising from inorganic processes, just like the origin of the Earth from the viewpoint of universal processes, or human evolution from animal life.

The new always clashes with the formidable probabilities of statistical laws which, under ordinary circumstances, amount practically to a certainty. This fundamental paradox—events arise without apparent cause—has major consequences for what we conceive as real, because our real is rooted in what happens, and what happens defies traceable origins despite its sudden emergence. That is where our “lifeline” of rationalism breaks: modern reason has become shipwrecked on the reef of the real [7].

Asymmetry and Chaos

Are we therefore led to a dead end? Certainly not if we now turn to the dynamics of non-equilibrium systems! The most fascinating system of this type is undoubtedly life. The dissipative structures highlighted by Ilya Prigogine [8] have definitively introduced bifurcation into our way of thinking about the future: a sudden, unexpected event that radically alters the space-time trajectory of any (sub)system. For Giuseppe Longo [9], historicity must be understood in terms of changes to the space of possibilities (phase space) as well as the role played by diversity in the structural stability of living systems and the role of rare events in

shaping history. On the stage of life, this places the “unpredictable” at the centre. Several studies have linked the thermodynamics of non-equilibrium systems (dissipative structures, chaotic systems) to asymmetric distributions. Overall, complexity studies rest on the role of such distributions in non-equilibrium steady states, entropy production, and symmetry breaking¹¹.

But we must be precise about what we are discussing. First, these systems—far from thermodynamic equilibrium—show particular behaviour. Their structure directly results from energy flowing through them. If the energy flux dries up, the structure disappears. In Northern Europe, we enjoy a temperate climate compared to North America at the same latitudes, thanks to the Gulf Stream—which climatologists now fear may end. Indeed, if this ocean circulation stopped or reversed, we would face the same harsh winters as New York.

Another trait is that these systems are extremely sensitive to even weak fluctuations. Small variations can suddenly spread through the entire system, steering it in a totally unexpected direction. This is the opposite of equilibrium systems, where a small environmental change is damped by the system’s components. A well-known effect in innovation networks concerns their resilience: a well-established network with many partners is less resilient than a “fragile” (but more agile) one, as it instinctively rejects the introduction of a new partner (here playing the role of perturbation or fluctuation) whose ideas and innovation seem out of frame or irrelevant.

Thus, near equilibrium, rare fluctuations cancel out statistically, and the system stays predictable around its mean. Far from equilibrium, continuously injected energy (e.g., wind for turbulence, food for life, innovation for networks...) amplifies these rare deviations, making them decisive for system evolution. Imagine a boat: in calm (equilibrium), rare waves do not matter; in a storm (non-equilibrium), an exceptional wave—a rogue wave—can change everything.

In living systems (non-equilibrium maintained by metabolism), rare genetic mutations or mass extinctions drive evolution by altering natural selection. They enable major adaptive leaps, such as new species emerging, because the system is far from equilibrium and sensitive to extreme deviations. Thus, life’s history often depends on these amplified “accidents” rather than gradual changes.

Take cystic fibrosis: this genetic disease stems from the deletion of three base pairs in the gene coding for the protein that transports chloride ions across cell membranes. Human DNA has about

¹¹ Place a perfectly sharpened pencil flat on a smooth round table. Theoretically symmetric, it can roll indifferently left or right. In practice, a tiny imperfection makes it tip to one specific side—the rotational symmetry breaks spontaneously, creating a privileged direction without apparent external cause.

3 billion nucleotide base pairs. Statistically, what weight should be given to a “mistake” of 1/1,000,000,000? If we now consider the living conditions of affected patients, our viewpoint will certainly be entirely different.

In conclusion, non-equilibrium systems like life or turbulence do not trend towards a stable average state but constantly evolve through energy or matter exchanges with their environment. In these systems, rare events—low-probability but high-impact—often dominate overall behaviour, unlike equilibrium systems where the average suffices.

Prigogine’s foundational works—from his 1955 statistical mechanics of irreversible processes to his 1977 synthesis with Nicolis—show how dissipative structures emerge in systems far from equilibrium—like living cells or oscillating chemical reactions—by channelling energy fluxes towards dynamic order, well beyond static equilibrium [10-12]. Asymmetry plays a central role there: Lefever and Prigogine’s 1968 paper highlights instabilities that break symmetry, amplifying tiny biases into non-Gaussian asymmetric distributions that defy “bell curve” predictability and spawn unexpected temporal rhythms and spatial patterns [11].

Blum [2] wields “time’s arrow” not as evolution’s architect, but as its stern sentinel: entropy swells universally, yet local eddies of order—negentropy—arise through energy fluxes, yielding complexity without defying the grand dissipation. Blum describes life as a dynamic steady state that is not thermodynamic equilibrium (where fluxes vanish) but a stable non-equilibrium regime with continuous dissipation. So, Blum and Prigogine converge on self-organization via irreversible fluxes.

Thus, chaos enters through fluctuating intermittency, where Glansdorff and Prigogine (1971) stress stability within flux, and rare events—sudden bifurcations—redirect trajectories unpredictably, echoing the improbable leaps of life highlighted in Prigogine’s 1967 thermodynamic essay on biology [13,14]. Together, asymmetry, chaos and rare, unpredictable fluctuations divert entropy’s course towards creative evolution.

Cross-Field Perspectives

We shall now try to understand how a system shifts from near-equilibrium to far-from-equilibrium states, and what happens during this transition. For this, we shall consider a very simple system: a protein molecule in solution subjected to an electric field between two electrodes—the anode (positive pole) and the cathode (negative pole). Most proteins carry a negative charge at neutral

pH. This simulation is adapted from Pérez-Cárdenas [15]. These more or less charged molecules will thus tend to migrate towards the anode through opposite-charge attraction ¹² (Fig. 4).



Figure 4. Migration of a charged particle in an electric field

In this system, two distinct forces act on the molecule. First, it is subject to Brownian motion: depending on temperature, water molecules jostle it in a disorderly manner, with speed and kinetic energy¹³ that increase with temperature. Think of bumper cars: no preferred direction exists, and molecules move randomly in solution as a result of these constant collisions.

The second force comes from the electric field the protein experiences because it carries an electric charge (negative in our example). This one has a defined direction: the protein migrates towards the opposite pole, and its strength ($F = qE$) depends directly on field intensity and the charge q carried by the protein.

For our demonstration, we shall consider motion along just one dimension (here the horizontal x -axis), although thermal agitation affects all three spatial dimensions, of course. The simulation experiment starts at $t = 0$ and runs for 10 time units without specifying whether these correspond to milliseconds, seconds, or otherwise; the number of molecules is fixed at 1 million (1,000,000). After this time $t + 10\Delta t$, we observe the particle position distribution along the x -axis.

¹² This is the principle of electrophoresis, used to separate proteins by electric charge—from the Greek “phórēsis,” meaning to transport, migrate.

¹³ This is quantifiable—called the thermal bath—and equals $K_B T$, the product of Boltzmann’s constant and absolute temperature in Kelvin.

First, a position distribution curve is generated, and more precisely, the cumulative frequency curve is added: what proportion of molecules has reached distance x . What varies in our simulation experiment is the electric field strength E ; temperature remains constant. We shall call $\beta = \frac{qE}{K_B T}$ the ratio of these two forces. It equals zero when no field is applied, and rises to 3 here when the directional force from the field on the protein is three times stronger than the random thermal agitation.

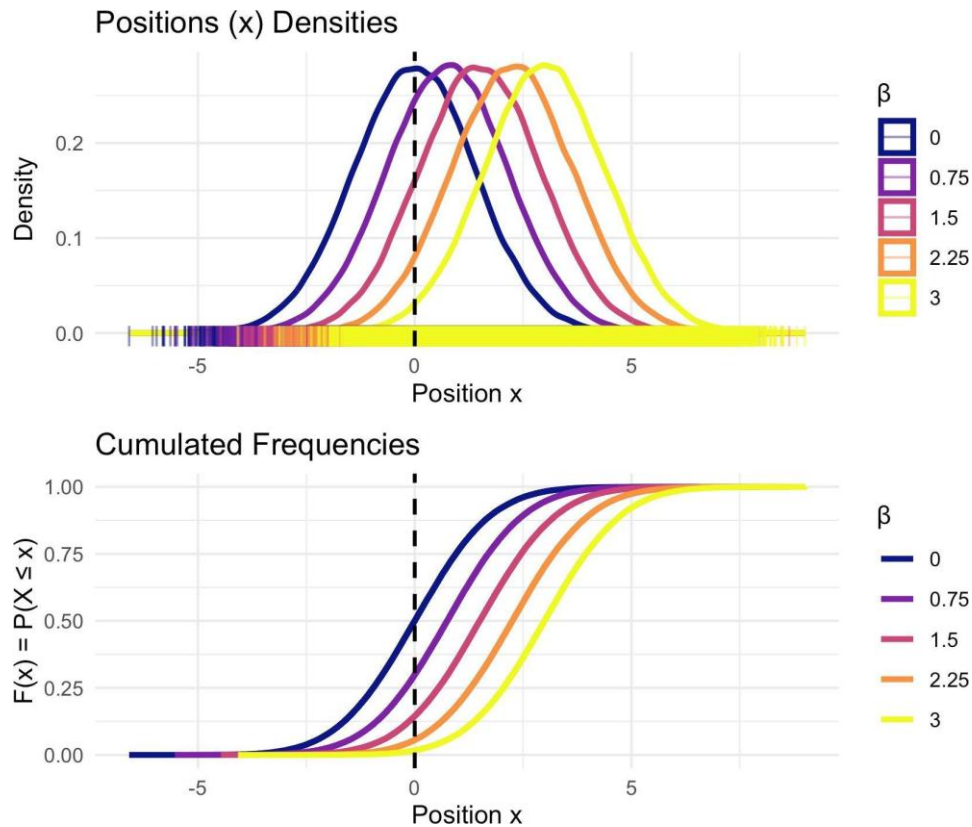


Figure 5. Evolution of skewness as the electric field strength increases

If molecules are subject only to thermal agitation ($\beta = 0$), final positions (blue curves) follow a perfectly symmetric “bell curve” distribution with zero mean: as many molecules migrate left as right. If the electric field is now non-zero ($\beta > 0$), more molecules migrate rightwards under this force: the stronger the field ($\beta \gg 0$), the more the distribution shifts towards positive x -values and becomes asymmetric. The shift of the cumulative frequency curves shows this: just look at the mean position ($F(x) = 0.50$) that half of the molecules have reached. With no field, this equals zero (symmetric displacements on either side of the starting point); for $\beta = 3$, it reaches about $x = 3$, meaning that 500,000 molecules (half) reach a distance of +3 after 10 time steps.

Here, counter-migrating molecules dwindle, acquiring an increasingly random character. These rare trajectories reveal non-linear instabilities. The practical impact of these few “delayed” molecules matters, as they affect the global dynamics. As noted earlier, this technique separates molecules by charge. Yet real physical systems have constrictions—local narrowings in migration channels where the electric field intensifies sharply, altering mean speeds through heightened trapping.

For $\beta > 1$, this trapping halts net flux. Beyond $\beta > 1.5$, convection emerges, spawning chaotic swirls near traps that release molecules not continuously but in bursts—boosting mass transport by up to tenfold: molecules then clearly separate and are detected in packets, raising the technique’s resolution through turbulent (not laminar) flow. Thus, equilibrium conditions end as soon as $\beta > 1.5$, with chaotic convection: turbulence appears.

We see, then, that these rare events alter the mean speed, increase trapping, and paradoxically improve molecule separation. In biomedical applications, these abnormal flows concentrate analytical targets, making analyses more sensitive despite the underlying dynamic complexity. Indeed, charged particles often serve as biomarkers (proteins, cancer exosomes). Normal trajectories ($\beta < 1$) disperse them uniformly in the channel. At $\beta > 1$, selective trapping preferentially holds larger or more polarizable ones at constrictions, whereas smaller ones pass. Chaotic convection ($\beta > 1.5$) then creates concentrated bursts: instead of a diffuse flow, biomarkers arrive in intense peaks at detectors, boosting the analytical signal by a factor of ten.

Thus, these rare events transform passive migration into enriched dynamic separation, enhancing trace cancer detection despite a lower mean flux.

These macroscopic flow changes—selective trapping, concentrated bursts, and signal enhancement—stem directly from the linear non-equilibrium regime initiated by rare negative trajectories at $\beta = 1$. In the linear regime ($\beta < 1$), modelling¹⁴ predicts straight trajectories and uniform, laminar flow without inversions or chaos. At $\beta > 1$, nonlinear trapping breaks this linearity: microscopic fluctuations (negative trajectories) propagate into collective instabilities, creating trapping and convection that are impossible at equilibrium.

Without nonlinearity, no bifurcation to chaos occurs; rare events are the observable precursors of this shift to active dynamics that enrich biomarker separation. Thus, linear modelling fails precisely

¹⁴ Hückel–Smoluchowski.

because it ignores these effects, confirming that macroscopic non-equilibrium arises from microscopic imbalance.

Conclusion

Where have we landed? From the erratic behaviour of simple gas particles, we have drifted towards the history of events that make History—unexpected events weaving the fabric of our daily reality. For living systems, inherently far from equilibrium, bifurcations set the clocks. Every new second, every moment, holds the potential to act as a starting point for time’s arrow, since rare events decisively shape the future, and thus our history [9].

The apparent level of complexity of the analysed subsystems matters little: from atoms (or subatomic particles) to the universe, via galaxy clusters, every system is a system of out-of-equilibrium systems where outsiders hold sway over their destiny. Rare events are not “anecdotal” or mere “news scoops”: they drive the creative forces of our worlds. *What comes into the world to disturb nothing deserves neither regard nor patience* [16].

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Appendix

A new model was designed, based on a 32×32 matrix whose $n = 1024$ cells can host from 0 to N particles. As before, a random—thus non-uniform—initial distribution is set for N particles ($32 \leq N \leq 4096$). This initial distribution then evolves (at constant N) over a few (8) time steps according to a simple rule deliberately mimicking gravity: the probability that a given cell hosts additional particles at any moment is weighted by the square of the number of particles it hosted at the previous instant. This results in faster or slower particle clustering, depending on the initial partition and the particle-to-cell ratio (N/n).

Here, the thermodynamic entropy calculation equations are as follows: for a 32×32 matrix containing N particles distributed non-uniformly, we use Boltzmann's entropy formula with the Stirling approximation.

Stirling approximation for a large number of particles n :

$$\ln(n!) \approx n \ln n - n$$

Total system entropy: entropy S is proportional to the logarithm of the number of micro-states Ω :

$$S = k_B \ln \Omega$$

Where

$$\Omega = \frac{N!}{\prod_{i=1}^{1024} n_i!}$$

with

$$N = \sum_{i=1}^{1024} n_i \text{ total number of particles in the matrix}$$

n_i : number of particles in cell i .

Using the Stirling approximation, entropy becomes:

$$S = N \ln N - N - \sum_{i=1}^{1024} [n_i \ln n_i - n_i]$$

Simplifying (since $N = \sum_{i=1}^{1024} n_i$):

$$S = N \ln \ln N - \sum_{i=1}^{1024} * ni \ln ni$$

High entropy (e.g., uniform distribution) indicates that particles are spread homogeneously (maximum disorder), whereas low entropy (e.g. concentrated distribution) indicates that they are clustered in a few cells (high order). These equations quantify the entropy decrease as particles aggregate, reflecting the growing order in the corresponding system. Figures 4 and 5 show how particle spatial distributions and the resulting entropy evolve over time for $N = 512$ and $N = 4096$, respectively.

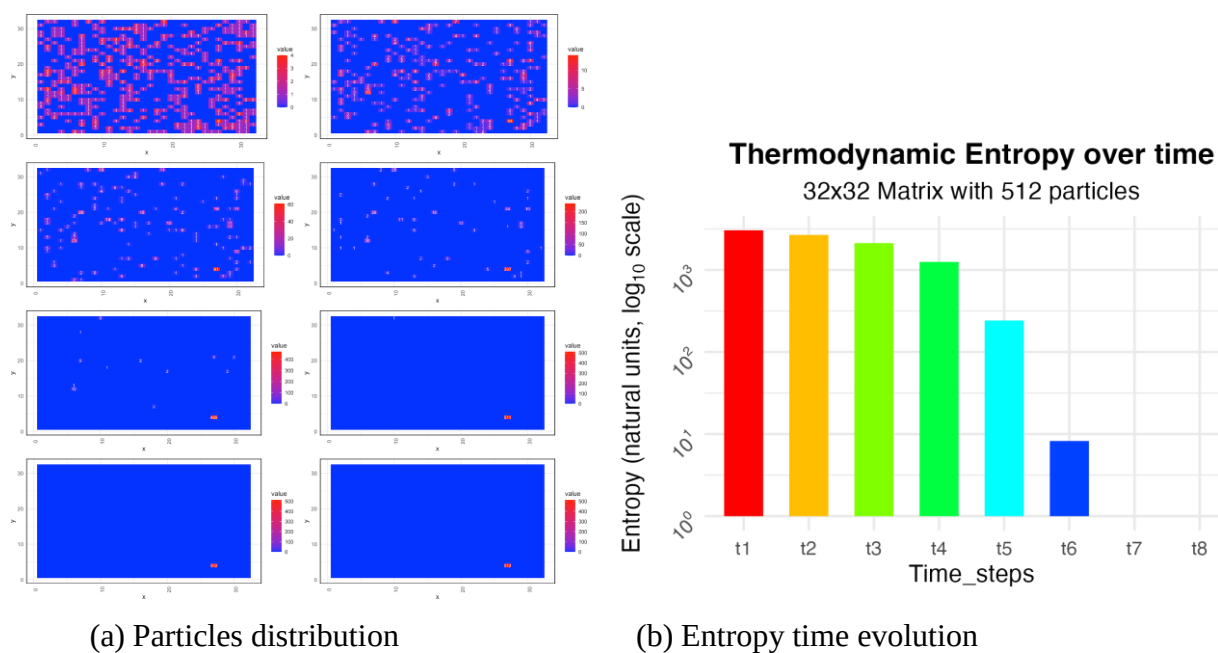


Figure 6. Model $\frac{N}{n} = 0,5$

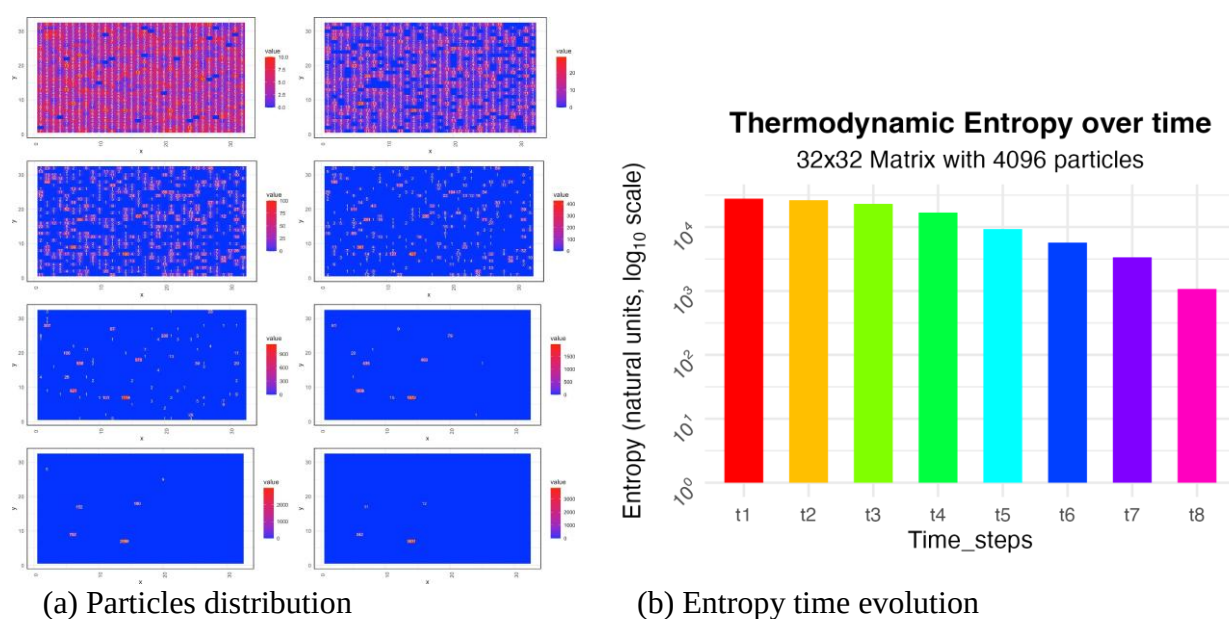


Figure 7. Model $\frac{N}{n} = 4$

Figures 6a and 7a show, over the 8 time steps, how many particles each cell hosts. Figures 6b and 7b present the corresponding entropy evolution over time, on a logarithmic scale. A zero value, as seen at $t = 7$ and $t = 8$ for $N = 512$, indicates that all particles have migrated to a single cell.

We observe that in “crowded” conditions ($N = 4096$), cluster formation is slower and, consequently, entropy decrease is proportionally weaker compared to “sparse” conditions ($N = 512$), where particles quickly cluster into one cell, giving birth to a tiny, rare, low-entropy world. Note that “fast” and “slow” here are purely qualitative, as are the time steps, which do not correspond to any defined temporal span.

Focusing now on the fate of an initially very dense subsample ($N = 4096$) of the universe, we observe that it evolves towards four small worlds whose sizes span three log units (Fig. 8): in practice, the smallest hosts 11 particles, whereas the largest contains nearly 4,000.

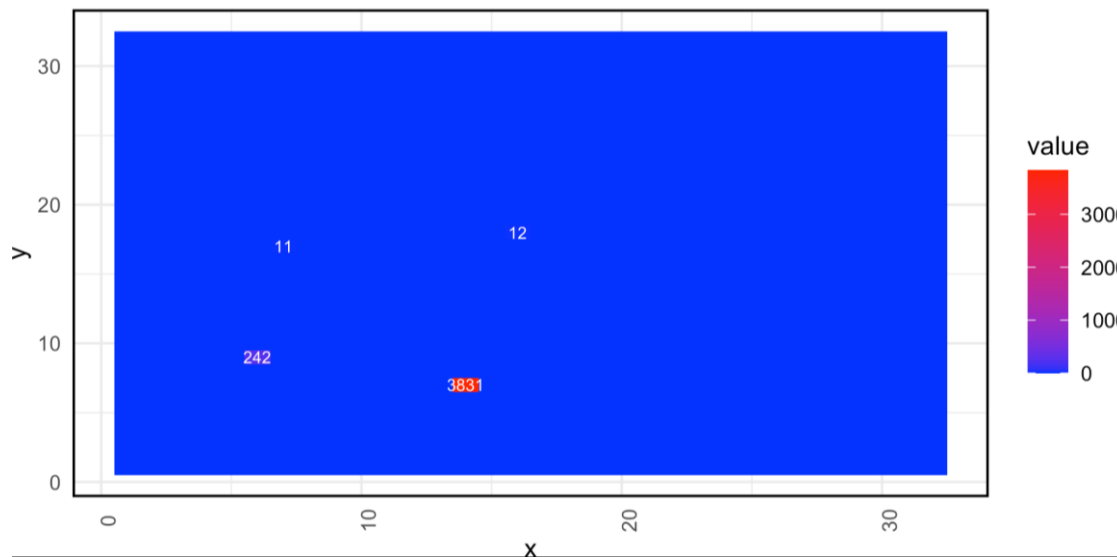


Figure 8. Particles distribution at $t = 8$ for $N = 4096$

These four “worlds” will likely now evolve differently, as a result of distances and internal particle interactions, which must depend on their respective sizes and mutual separations. One is tempted to see fractal-like properties linking the behaviour of a low-interaction particle with its environment to that of a denser entity—that is to say, composed of more individuals. The crucial point is that interactions—both in number and in nature—between constituent elementary entities truly drive the dynamics and determine the fate of these subsystems.